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Astronomy 406: Galaxies and the Universe
Placement Exam
August 2015

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1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
2. Write you name above. Turn in these pages and your study sheet if you have one.
3. Show all of your work and indicate clearly your final answer! A correct final answer may not receive credit if no work is shown.
4. Budget your time! Don't get stalled on any one question.
5. The exam is open book and open notes.
6. For your reference there are constants and equations listed below.
7. The total number of points on the exam is 100.

Possibly Useful Constants

Astronomical Unit: $1\,\mathrm{AU} = 1.5 \times 10^{11}\,\mathrm{m}$.
parsec: $1\,\mathrm{pc} = 3.1 \times 10^{16}\,\mathrm{m} = 2.1 \times 10^{5}\,\mathrm{AU}$
gravitational constant : $G = 6.7 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} = 4.5 \times 10^{-3}\ \mathrm{pc\ km^2\,s^{-2}}\ M_\odot{}^{-1}$
speed of light: $c = 3.0 \times 10^{8}\ \mathrm{m\,s^{-1}} = 3.0 \times 10^{5}\ \mathrm{km\,s^{-1}} = 1.023\ \mathrm{pc\,Myr^{-1}}$
Stefan-Boltzmann constant: $\sigma = 5.7 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$
Wien's Law constant: $b = 2.9 \times 10^{-3}$ m K
Planck's constant: $h = 6.6 \times 10^{-34}$ J s $= 4.1 \times 10^{-15}$ eV s
electron Volt: 1 eV $= 1.6 \times 10^{-19}$ J
Boltzmann constant: $k = 1.4 \times 10^{-23}$ J K^{-1} $= 8.6 \times 10^{-5}$eV K^{-1}
mass of the proton: $m_p = 1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}/c^2$
mass of the neutron: $m_n = 1.675 \times 10^{-27}\,\mathrm{kg} = 939.6\,\mathrm{MeV}/c^2$
mass of the electron: $m_e = 9.1 \times 10^{-31}\,\mathrm{kg} = 0.511\,\mathrm{MeV}/c^2$
solar mass: $M_\odot = 2.0 \times 10^{30}$ kg
solar (total) luminosity: $L_\odot = 3.8 \times 10^{26}\,\mathrm{Watt} = 3.8 \times 10^{33}\ \mathrm{erg\,s^{-1}}$
present age of Sun $\sim 5 \times 10^{9}$ yr
Hubble constant (present-day value): $H_0 \simeq 73$ km s^{-1} Mpc^{-1} $= 2.3 \times 10^{-18}$ s^{-1}
Hubble length: $d_H = c/H_0 = 4.1 \times 10^{3}$ Mpc
Hubble time: $t_H = 1/H_0 = 13.4$ Gyr
present critical density: $\rho_{\mathrm{crit},0} = 3H_0^2/8\pi G = 1.0 \times 10^{-29}$ g cm^{-3} $= 1.5 \times 10^{11}\ M_\odot$ Mpc^{-3}
present CMB temperature: $T_0 = 2.725 \pm 0.001$ K
present number density of CMB photons: $n_{\gamma,0} = 411$ cm^{-3}
baryon-to-photon ratio: $3.4 \times 10^{-10} \leq \eta \leq 6.9 \times 10^{-10}$

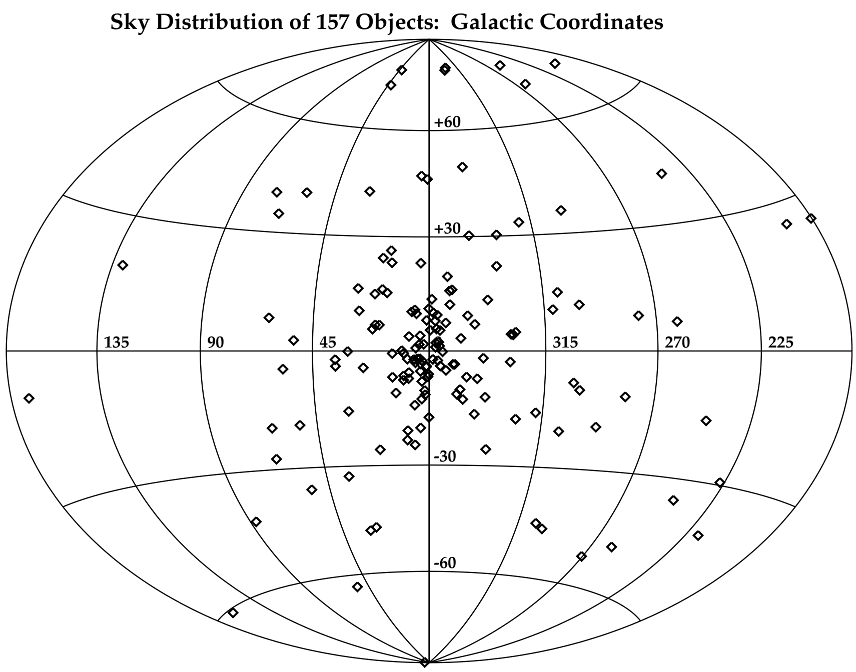
Possibly Useful Formulae

Note: symbols may have different meanings in different equations!

$v_{\rm circ}^2 = GM/r$	$a^3 = GMP^2/4\pi^2$	
$\lambda f = c$	$E_\gamma = hf = hc/\lambda$	$p = h/\lambda$
$E_n = -13.6\ {\rm eV}/n^2$	$E = mc^2$	
$\lambda_{\rm max} T = 2.9 \times 10^{-3}\ {\rm m} \cdot {\rm K}$		
$F = \sigma T^4$	$\varepsilon = 4\sigma T^4/c$	$F = L/4\pi r^2$
$\vec{v} = H\vec{r}$	$\vec{r}(t) = a(t)\,\vec{r}_{\rm today}$	$H = \dot{a}/a$
$(\dot{a}/a)^2 = 8\pi G\rho/3 - K/a^2$	$\ddot{a}/a = -4\pi G/3(\rho + 3P/c^2)$	$d(a^3\rho) + p\ d(a^3) = 0$
$\Omega_i = \rho_i/\rho_{\rm crit}$	$\rho_{\rm crit} = 3H^2/8\pi G$	
$P = w\rho$	$\rho_w = \rho_{w,0}\ a^{-3(1+w)}$	
$\rho_{\rm m} = \rho_{\rm m,0} a^{-3}$	$\rho_{\rm r} = \rho_{\rm r,0} a^{-4}$	
$\lambda_{\rm obs} = \lambda_{\rm em}/a_{\rm em}$	$a = 1/(1+z)$	$T = T_0/a$

1 Questions

1. *Mystery Objects.* A certain class of objects appears in the figure at right, which displays the positions on the sky, plotted in Galactic coordinates. Each open diamond represents the location of one of these objects, of which 157 are known.



   (a) **[10 points]** Based on the sky distribution, are these objects dominantly Galactic or extragalactic? Briefly explain your reasoning.

   (b) **[10 points]** Based on the sky distribution, does our sample span the entire region occupied by these objects, or just the local neighborhood? That is, if they are Galactic, are we seeing them throughout the entire Galaxy? If they are extragalactic, are we seeing them beyond the local supercluster? Briefly explain your reasoning.

   (c) **[5 points]** Finally, speculate as to what these mystery objects might be, and briefly justify your answer.

2. *Rotation Curves.* For some galaxies, the innermost rotation curve is linear, i.e., the rotation speed v grows with radius r as

$$v(r) = \alpha r \tag{1}$$

where α is a constant. This relationship holds for $r \leq r_{\rm max}$, that is, out to some radius $r_{\rm max}$.

   (a) [**5 points**] Assuming circular orbits, find an expression for the orbit *period* as a function of r for $r \leq r_{\rm max}$, in terms of α and possibly physical constants.

      - Sketch a plot of period versus radius.
      - Briefly comment on the nature of the motion.

   (b) [**10 points**] Still assuming circular orbits, find the mass density ρ as a function r for $r \leq r_{\rm max}$, in terms of α and possibly physical constants.

      - Sketch a plot of ρ versus r.
      - Briefly comment on the physical significance of the result.

   (c) Dark matter models suggest that the density of dark matter in the inner regions of galaxies should grow as

$$\rho_{\rm dm}(r) = \rho_{\rm c}\frac{r_{\rm c}}{r} \tag{2}$$

   i.e., $\rho_{\rm dm} \propto 1/r$, where both $\rho_{\rm c}$ and $r_{\rm c}$ are constants.

      i. [**5 points**] Find the rotation curve pattern this gives.

      ii. [**5 points**] Can we combine dark matter with this density structure with ordinary (baryonic) matter in order to reproduce the observed rotation curve in eq. (1)?
      If so, describe but do not calculate the baryonic matter density pattern needed.
      If not, explain why not.

3. *Cosmic Expansion.* Consider a universe that is geometrically flat, i.e., Euclidean, and which contains the following:

- non-relativistic matter, with present density parameter $\Omega_{\rm m}$
- relativistic matter, with present density parameter $\Omega_{\rm r}$
- a cosmological constant, with present density parameter Ω_Λ

Let the present value of the Hubble parameter be H_0.

(a) [**10 points**] For the above universe, write down an expression for the cosmic density, $\rho(a)$, as a function of scale factor a, using *only* the parameters above, and possibly physical constants.

(b) [**5 points**] Use your result from part (a) to find $\Omega_{\rm r}(a)$, and $\Omega_{\rm m}(a)$, the cosmic density parameters of matter and radiation respectively, as a function of a.

(c) [**5 points**] Use your result from part (b) to show that that $\Omega_{\rm r} \to 1$ as $a \to 0$. Briefly comment on this result and interpret it physically.

(d) [**5 points**] Matter-radiation equality is said to occur when the matter and radiation densities of the universe are equal. Find an expression for the scale factor $a_{\rm eq}$ and redshift $z_{\rm eq}$ of the matter-radiation equality epoch.

4. *Big-Bang Nucleosynthesis: Primordial Helium*

(a) [**6 points**] For cosmic times ≤ 1 sec, to an excellent approximation there are no bound nuclei in the universe, but only neutrons and protons. At these times, the Weak reactions $n + \nu_e \leftrightarrow p + e^-$ and $n + e^+ \leftrightarrow p + \bar{\nu}_e$ are rapid, and these interconversions lead to an equilibrium.

Treat the neutron and proton as a 2-state system (called the "nucleon"), with the proton as the ground state, and the neutron as the excited state. The nucleon is spin-1/2. Show that for such a system, equilibrium at temperature T leads to a neutron-to-proton ratio

$$\left(\frac{n}{p}\right)_{\text{eq}} = e^{-\Delta mc^2/kT} \tag{3}$$

where $\Delta m = m_n - m_p = 1.3$ MeV is the neutron-proton mass difference.

(b) [**4 points**] Find the equilibrium n/p ratio in the limit $kT \to \infty$ and $kT \to 0$, and briefly comment on the physical significance of your results.

(c) [**6 points**] At some temperature T_f, the Weak reactions freeze out, and after this the n/p ratio is essentially fixed to the equilibrium value $(n/p)_f$ at T_f (you may ignore free neutron decays).

Once nuclear reactions commence, essentially all neutrons eventually are incorporated into ^{4}He. Show therefore that the resulting primordial helium mass fraction is

$$Y_p = \frac{\rho(^4\text{He})}{\rho_{\text{baryon}}} = \frac{2(n/p)_f}{1 + (n/p)_f} \tag{4}$$

where ρ is the mass density, and where outside of exponents you may take $m(^4\text{He}) \approx 4m_p$ and $m_n \approx m_p$. You may also assume all baryons are non-relativistic.

(d) [**4 points**] Find Y_p in the limit $T_f \to \infty$ and $T_f \to 0$. Briefly comment of the physical significance of each result.

(e) [**5 points**] Observations have determined that $Y_p \approx 1/4$. From this, and your results above, find the freezeout temperature T_f in Kelvin, and the corresponding freezeout redshift z_f.