Placement Exam 2013: Astronomical Techniques (414)

(25 POINTS) COORDINATE SYSTEMS

1. Two stars, denoted A and B, have apparent right ascension (α) and declination (δ) of date:

   $\alpha_A = 12^h\,50^m\,18.0^s \quad \delta_A = 59^d\,59^m\,58.0^s$

   $\alpha_B = 12^h\,50^m\,18.4^s \quad \delta_B = 60^d\,00^m\,02.0^s$

   a. What is the instantaneous scalar great-circle separation Δ between star A and star B on the plane of the sky (in seconds of arc) ? Assume that the small-angle approximation applies.

   b. An optical telescope, equipped with a CCD camera with an effective focal length $f = 10.726\text{ m}$, is used to image this double star system. The CCD frame is large enough to image both stars simultaneously. If the frame is tiled contiguously with CCD pixels of physical linear dimensions: $10\mu\text{m} \times 10\mu\text{m}$, what is the pixel separation of star A and star B in the CCD image ?

   c. If the telescope has an aperture diameter D, write down an expression that is proportional to the speed, or energy deposition rate, of the telescope-camera optics.

   d. Assume the double star has no discernible intrinsic motions. Which telescope mount type, for an observatory at a northern mid-latitude, would cause the double star to rotate in orientation in the focal plane over the course of a night's observing and which telescope mount type would <u>not</u> lead to this field rotation ?

(25 POINTS) PROPAGATION

2. An astrophysical background source emits radiation with specific intensity I_0. An intervening gas cloud along the line of sight to the observer emits with a specific intensity I_s when it is optically thick. The radiative transfer equation (RTE) can be written in the form: $\frac{dI}{d\tau} = -I(\tau) + I_s$, where τ is optical depth, with $\tau = 0$ at the far-side of the intervening cloud, and $\tau > 0$ at the position of the observer.

   a. Show that the solution to the RTE is: $I(\tau) = I_0 e^{-\tau} + I_s\left(1 - e^{-\tau}\right)$.

   b. Write down the limiting solution for $I(\tau)$ at low optical depth when $I_0 \neq 0$.

c. If a molecular species in the intervening gas cloud has a spectral line at a certain rest frequency ν_0 and the condition $I_s < I_0$ applies, motivate from the form of the solution for (b) whether this line will be in absorption or emission relative to the adjacent continuum black-body emission.

(25 POINTS) PHOTON DETECTORS:

3. Consider a photon-counting detector, with Poisson statistics, that is being used for optical photometry. Assume that the detector has negligible read noise and dark current.

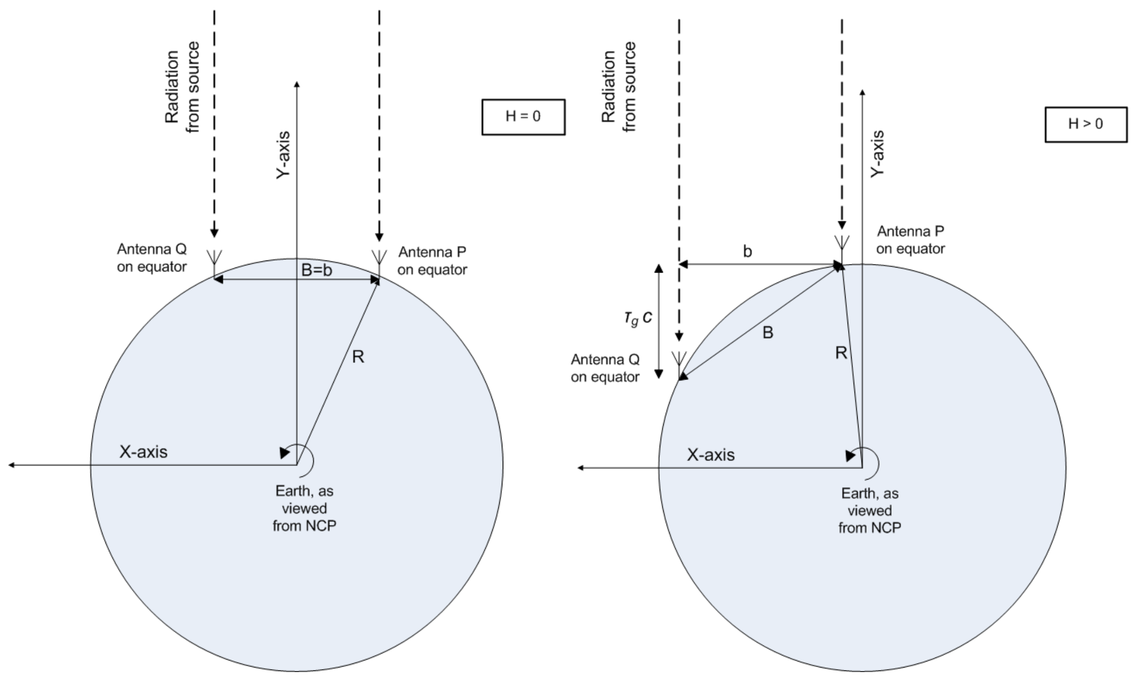
   a. If the photon count of the detector on a star is S what is the uncertainty σ_S in S?

   b. If the photon count on the sky background is *B*, then the instrumental magnitude of the star can be calculated as: $m = C - 2.5\log_{10}\left(\frac{S-B}{t}\right)$, where *C* is a zero-point instrumental constant (assumed exactly known) and *t* is the exposure time. Assuming the errors in photon counts *S* and *B* are statistically independent, what is the uncertainty σ_m in the instrumental magnitude m?

   c. In the limit of zero sky background *B,* show that σ_m is proportional to the inverse signal-to-noise ratio in *S*.

(25 POINTS) INTERFEROMETRY

4. A two-element interferometer (shown below) is constructed by building a baseline exactly east-west on the equator. Antenna *P* is positioned *B/2* km west of the local meridian and antenna *Q* is positioned *B/2* km east of the local meridian. The interferometer observes with a phase center at a fixed apparent right ascension $\alpha_{app} = \text{const.}$ and apparent declination of date $\delta_{app} = 0$.



a. Derive an expression for the geometric delay $\tau_g(H)$ of the interferometer baseline P-Q as a function of the local hour angle H (hours) at the mid-point of the array and the baseline separation B (km). Assume that the Earth rotates with a constant angular speed $\omega_E = 0.26$ rad h^{-1}.

b. Exploiting the vector relation $(\tau_g c)^2 + b^2 = B^2$ and the answer from (a), derive an expression for the (u,v) locus b=($u(H)$, 0) traced by the interferometer baseline P-Q in the u-v plane as a function of local hour angle H (hours) at the mid-point of the array and the baseline separation B (km). Assume that the observations are performed at a wavelength λ (m).

c. If the measured complex visibility $V(u,v)$ on the P-Q baseline takes the form $|V| = \gamma$ and $\arg(V) = \beta u$, where both (β, γ) are scalar constants, what can you deduce qualitatively about the form of the source brightness distribution ?
