ASTRONOMY PLACEMENT EXAM
ASTRONOMICAL TECHNIQUES: 2015

INSTRUCTIONS:

- Answer ALL four (4) questions. The exam is four pages long.
- For mathematical problems, always obtain an algebraic answer first, then proceed to an arithmetic answer if the problem requests a numerical result.
- For longer mathematical problems, please explain briefly what you are doing as a potential aid to graders. Even if the solution is wrong, you may be able to receive partial credit.
- Please specify your units; an answer without units will be penalized.
- Please write legibly; illegible answers cannot be graded reliably.
- You may use the exam paper for scratch work, but only answers in the test booklet will be graded.

TABLE OF CONSTANTS:

$M_{\mathrm{Sun}} = 1.99 \times 10^{33}$ g	$G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}
$R_{\mathrm{Sun}} = 6.96 \times 10^{10}$ cm	1 pc = 3.09×10^{18} cm
$L_{\mathrm{Sun}} = 3.83 \times 10^{33}$ erg s^{-1}	1 yr = 3.16×10^{7} s
1 AU = 1.50×10^{13} cm	$c = 3.00 \times 10^{10}$ cm s^{-1}
$k_{\mathrm{B}} = 1.38 \times 10^{-16}$ erg K^{-1}	$h = 6.63 \times 10^{-27}$ erg s
$m_{\mathrm{p}} = 1.673 \times 10^{-24}$ g = 938.3 MeV/c^2	$m_{\mathrm{e}} = 9.11 \times 10^{-28}$ g = 0.511 MeV/c^2
$m_{\mathrm{n}} = 1.675 \times 10^{-24}$ g = 939.6 MeV/c^2	$m_{\pi} = 139.6$ MeV/c^2
$e = 4.80 \times 10^{-10}$ esu = 1.60×10^{-19} C	1 eV = 1.60×10^{-12} erg
$\sigma_{\mathrm{SB}} = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}	1 J = 9.87×10^{-3} L Ä atm = 10^{7} erg
1 m = 10^2 cm = 10^6 µm = 10^9 nm	1 Å = 10^{-8} cm = 10^{-10} m = 0.1 nm
$M_{\mathrm{V(Sun)}}$ =+4.83 mag	$M_{\mathrm{B(Sun)}}$ =+5.48 mag
T_{Sun} = 5,778 K	$m_{\mathrm{V(Sun)}}$ =-26.5 mag
$R_{\mathrm{Earth}} = 6.37 \times 10^{3}$ km	$R_{\mathrm{Moon}} = 1.74 \times 10^{3}$ km
$M_{\mathrm{Earth}} = 5.97 \times 10^{27}$ g	$r_{\mathrm{Earth\text{-}Moon}} = 3.84 \times 10^{5}$ km
$M_{\mathrm{Jupiter}} = 1.90 \times 10^{30}$ g	$M_{\mathrm{Venus}} = 4.90 \times 10^{27}$ g
$R_{\mathrm{Jupiter}} = 7.14 \times 10^{4}$ km	$R_{\mathrm{Venus}} = 6.05 \times 10^{3}$ km
$H_0 = 100h$ km s^{-1} Mpc^{-1}	T = (0.290 cm) / $\lambda_{\max}$ K
$\tau^2 = \dfrac{4\pi^2}{G}\dfrac{a^3}{M_1 + M_2}$	1 W = 1 J s^{-1} = 10^7 erg s^{-1}

Placement Exam 2015: Astronomical Techniques (414)

(25 POINTS) RADIATION QUANTITIES

1. An optically-thick thermal blackbody emitter has a specific intensity
   $I_\nu(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$ W m^{-2} Hz^{-1} sr^{-1}.

   a. By setting: $\int_{\nu_1}^{\nu_2} I_\nu(\nu,T)\,d\nu = -\int_{\lambda_1=c/\nu_1}^{\lambda_2=c/\nu_2} I_\lambda(\lambda,T)\,d\lambda$ show that the specific intensity per unit wavelength is given by: $I_\lambda(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$ W m^{-3} sr^{-1}.

   b. Convert $I_\lambda(\lambda,T)$ obtained in (a) to photon form with units of photons s^{-1} m^{-3} sr^{-1}.

   c. Consider the thermal background emission from Earth's atmosphere, treated as a blackbody emitter at temperature T= 300 K with unit emissivity. From the expression derived in (b) show that the background photon flux density from the atmosphere is:
   $I_\lambda(\lambda_\mu, T=300) = 1.41\times10^{16}\,\lambda_\mu^{-4}\big/\left(e^{48/\lambda_\mu}-1\right)$ photons s^{-1} m^{-2} μm^{-1} arcsec^{-2}
   where λ_μ is the wavelength in microns.

   d. Calculate the background thermal emission from the atmosphere at $\lambda = 20\mu$m in units of photons s^{-1} m^{-2} μm^{-1} arcsec^{-2}. How does this influence the feasibility of far-infrared observations at this wavelength from ground-based observatories ?

(25 POINTS) IMAGING

2. An optical telescope of diameter D = 4.0 m is used at a camera focus of effective focal length $f = 11.5$ m. The camera focal plane is tiled with pixels that are each 15 µm square. The radius of the atmospheric seeing disk in this band at the observing site is $\theta_s = 1.0$ arcsec.

   a. Show that the plate scale at this focus is 18 arcsec/mm.

   b. Calculate the pixel area n_* in the camera focal plane occupied by an unresolved point source observed through the seeing conditions listed above.

c. Assume that the point source in (b) has a flux density F_* (W m^{-2}) and can be detected in a minimum integration time Δt_*. By equating energy per pixel in the focal plane determine the minimum integration time Δt_s that would be needed to detect an object of uniform brightness with an angular radius of 1 arcmin and a flux density $100F_*$ (W m^{-2}).

d. To reduce Δt_* should the focal length f be increased or decreased ? Explain your answer briefly.

(25 POINTS) CALIBRATION AND ERROR ANALYSIS

3. Consider CCD observations in the visual band. In the course of an observing sequence, an observer takes a bias frame $b(x,y)$, dark frame $D(x,y)$, normalized flat-field frame $F^n(x,y)$, and un-calibrated science frame $S^u(x,y)$, where (x,y) are the two-dimensional pixel coordinates on the CCD. The science frame is observed over an exposure time t_s and the dark frame over an exposure time t_d.

   a. Write down an expression for a calibrated science frame $S^c(x,y)$ in terms of $b(x,y)$, $D(x,y)$, $F^n(x,y)$, $S^u(x,y)$, t_s, and t_d.

   b. What specific CCD instrumental effects are corrected by the flat-field frame ?

   c. What is the physical origin of the dark current measured by the dark frame and how can it be reduced in practice ?

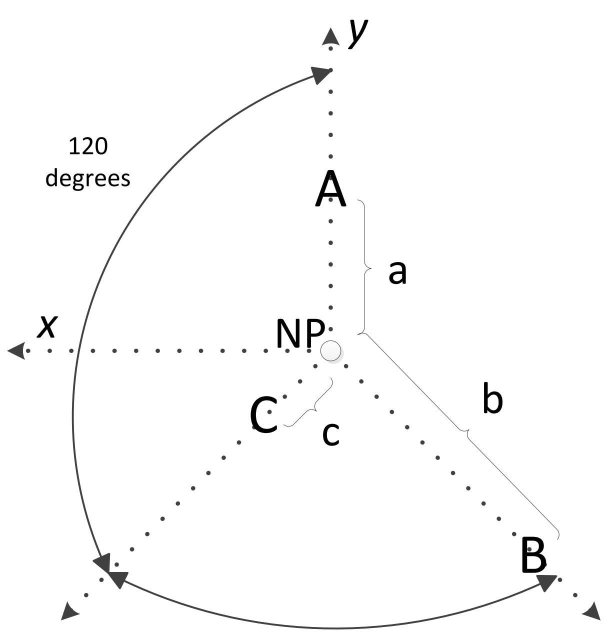
   d. A 1-dimensional profile of an astronomical source is extracted from the CCD and fit as a Gaussian with full-width half-maximum (FWHM) σ_{obs}. The measurement has an uncertainty $\sigma_{\text{obs}} \pm \varepsilon_{\text{obs}}$. Along this profile axis, the telescope point-spread response function (PSF) is also Gaussian with FWHM $\sigma_{\text{PSF}} \pm \varepsilon_{\text{PSF}}$. Then, the convolved observed source size σ_{obs} is connected to the true source size σ_{true} by the simple deconvolution relation: $\sigma_{\text{true}} = \sqrt{\sigma_{\text{obs}}^2 - \sigma_{\text{PSF}}^2}$.

      i. Treat ε_{obs} and ε_{PSF} as statistically uncorrelated and calculate the propagated uncertainty in the estimated value of σ_{true}.

      ii. For a fixed ε_{obs} and ε_{PSF} what asymptotic ratio of $\sigma_{\text{obs}}/\sigma_{\text{PSF}}$ maximizes the error in σ_{true}?

(25 POINTS) INTERFEROMETRY

4. A Y-shaped interferometer is built centered on the North Pole with three telescopes A, B, and C at the positions indicated in the Figure, with a = 10 m, b = 20 m, and c = 5 m. Neglect the curvature of the Earth and assume these are linear separations from the array center and that the interferometer lies in a flat plane. The interferometer observes at a wavelength $\lambda = 10\mu\mathrm{m}$ and observes a source directly overhead at the North Celestial Pole. Axis y lies along the prime meridian and the x-axis is tangent to the line of longitude 90 degrees (East) at the time indicated in the Figure.



a. Write down the set of non-redundant *(u,v)*-vector spacings sampled instantaneously by the interferometer at the time indicated in the Figure.

b. What is the maximum angular resolution of the interferometer (in milliarcsecond) ?

c. Draw a figure showing the *(u,v)*-tracks sampled by the interferometer in a total-intensity full synthesis observation over 12 hours duration. Label the axes numerically. {*Hint: V(u,v) also samples V(-u,-v) in this case*}.