Astronomy Placement Exam 2017
Solar System and Interstellar Medium (405)
Aug 25, 2017 — 2:00–4:00 PM

1. **Total Eclipse! [20 pts].** The Earth orbits the Sun with a semimajor axis of 1 AU and an eccentricity of 0.0167. The Moon orbits the Earth with a semimajor axis of 3.84×10^5 km and an eccentricity of 0.055. In both cases the semimajor axis is for the *relative* orbit, with the major body held fixed.

   (a) [**8 pts**] Calculate the minimum and maximum angular diameters of the Sun and Moon, as seen by an imaginary observer at the center of the Earth. Show that total solar eclipses (where the Moon completely obscures the Sun) are possible.

   (b) [**8 pts**] Estimate the speed of the Moon's umbral shadow through space during an eclipse, and compare it with the Earth's rotational speed at the equator. Explain why the shadow moves from west to east relative to the Earth's surface.

   (c) [**4 pts**] If the Moon's umbra is 100 km in diameter, estimate the duration (in minutes) of totality experienced by an observer on Earth's equator.

2. **Moment of Inertia [20 pts].** The moment of inertia is defined as

$$I = \int_V a^2 \, dm \ ,$$

   where a is the distance from the axis of rotation and dm is an infinitesimal mass element. In a spherical coordinate system we can define $a = r \sin\theta$.

   (a) [**5 pts**] Calculate the moment of inertia of a sphere with density ρ_1 and radius R_1. Note that

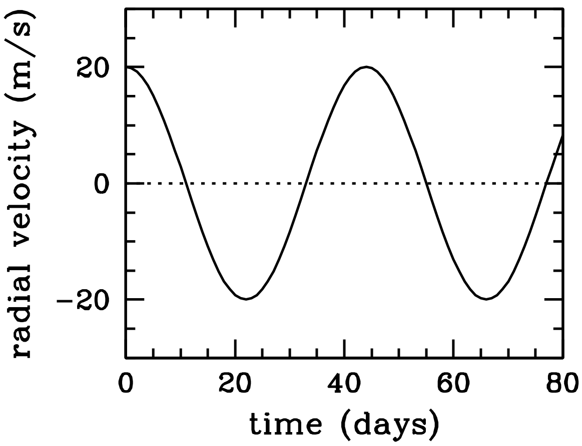
$$\int \sin^3\theta \, d\theta = \frac{1}{12}\left[\cos(3\theta) - 9\cos\theta\right] + \mathcal{C} \ .$$

   (b) [**5 pts**] Calculate the moment of inertia of a sphere with an empty (hollow) core of radius R_1, a density outside the core of ρ_2, and an outer radius of R_2.

   (c) [**5 pts**] Calculate the moment of inertia of a two-component sphere with a core of density ρ_1 and radius R_1, and an overlying mantle with density ρ_2 and outer radius R_2. Show that this case can be derived by superposition of cases (a) and (b).

   (d) [**5 pts**] By what factor does the moment of inertia change when going from a constant-density model of the Earth (R=6400 km, ρ=5500 kg m^{-3}) to a two-component model (R_1=3600 km, R_2=6400 km, ρ_1=10000 kg m^{-3}, ρ_2=4500 kg m^{-3})?

3. **Exoplanet Properties [20 pts].** A star with mass equal to the Sun's is observed to exhibit a perfectly sinusoidal variation in its radial velocity, with a period of 44 days and an amplitude of 20 m s^{-1} (see figure).



(a) **[4 pts]** Explain why this type of variation is indicative of a planetary companion. What would be the orbital period of the planet?

(b) **[8 pts]** Derive the minimum mass for the exoplanet that causes these variations. You may assume the planet's mass is much less than the star's. Express your answer in Jupiter masses ($M_J = 1.9 \times 10^{27}$ kg).

(c) **[8 pts]** For what range of orbital inclinations (with 90° being edge-on) would a transit be observable, in which the planet's disk blocks (occults) a portion of the star's disk? You may assume the radius of the star is equal to the Sun's and that the density of the planet is equal to Jupiter's (1300 kg m^{-3}).

4. **Radiation Pressure [20 pts].** Consider a spherical dust grain, located at a distance of 1 AU from the Sun, with a radius of 600 nanometers and a density of 3000 kg m^{-3}.

(a) **[7 pts]** In the absence of gravity, estimate the acceleration of the grain due to radiation pressure. Assume that the solar radiation is completely absorbed.

(b) **[4 pts]** Calculate the gravitational acceleration of the grain due to the Sun.

(c) **[4 pts]** Calculate the ratio of the radiation pressure force to the gravitational force. Which force is larger? Does the ratio depend on distance from the Sun?

(d) **[5 pts]** For grains of this assumed density, determine the grain radius at which the radiation pressure and gravitational forces exactly balance.

5. **Escaping Atmosphere [20 pts].** A star with similar mass and luminosity to the Sun is orbited by a giant planet with a radius of 70 000 km which is observed to transit in front of the star every 10 days.

(a) **[10 pts]** Estimate the equilibrium temperature of the planet by assuming it rotates rapidly and has an albedo of 0.3.

(b) **[10 pts]** Hydrogen (H_2) is observed to be escaping from the atmosphere of the planet. Derive an upper limit to the mass of the planet based on this observation. Express your result in units of the Earth's mass.