ASTRONOMY QUALIFYING EXAMINATION
STELLAR ASTROPHYSICS
August 19, 2011, 1:00 – 3:00 PM

INSTRUCTIONS:

- Answer ALL questions.
- Read a problem carefully. Pause and think before you attempt to solve it.
- Always obtain an *algebraic* answer first; then and only then an arithmetic one. BUT: if asked for, numbers are just as important as algebraic answers; do not skip them!
- Specify your units. An *arithmetic* answer without units is a *wrong* answer.
- Do NOT rename symbols given to you in a problem.
- Explain *briefly* what you are doing; that way you *may* get credit even if your solution is wrong. *If a grader cannot understand your reasoning, he/she may give you no credit*.
- Write LEGIBLY; illegible answers receive NO credit.
- You may use the back of the exam pages for scratch work.

TABLE OF USEFUL CONSTANTS:

$M_{\rm Sun} = 1.99 \times 10^{33}$ g	$G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}
$R_{\rm Sun} = 6.96 \times 10^{10}$ cm	1 pc = 3.09×10^{18} cm
$L_{\rm Sun} = 3.83 \times 10^{33}$ erg s^{-1}	1 yr = 3.16×10^{7} s
1 AU = 1.50×10^{13} cm	$c = 3.00 \times 10^{10}$ cm s^{-1}
$k_{\rm B} = 1.38 \times 10^{-16}$ erg K^{-1}	$h = 6.63 \times 10^{-27}$ erg s
$m_{\rm p} = 1.673 \times 10^{-24}$ g = 938.3 MeV/c^2	$m_{\rm e} = 9.11 \times 10^{-28}$ g = 0.511 MeV/c^2
$m_{\rm n} = 1.675 \times 10^{-24}$ g = 939.6 MeV/c^2	$m_{\pi^-} = 139.6$ MeV/c^2
$e = 4.80 \times 10^{-10}$ esu = 1.60×10^{-19} C	1 eV = 1.60×10^{-12} erg
$\sigma_{\rm SB} = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}	1 J = 9.87×10^{-3} L $\rightleftarrows$ atm = 10^7 erg
1 m = 10^2 cm = 10^6 μm = 10^9 nm	1 Å = 10^{-8} cm = 10^{-10} m = 0.1 nm
$M_{\rm V(Sun)} = +4.83$ mag	$M_{\rm B(Sun)} = +5.50$ mag
$T_{\rm Sun} = 5{,}778$ K	$m_{\rm V(Sun)} = -26.5$ mag
$R_{\rm Earth} = 6.37 \times 10^{3}$ km	$R_{\rm Moon} = 1.74 \times 10^{3}$ km
$M_{\rm Earth} = 5.97 \times 10^{27}$ g	$r_{\rm Earth\text{-}Moon} = 3.84 \times 10^{5}$ km
$M_{\rm Jupiter} = 1.90 \times 10^{30}$ g	$M_{\rm Venus} = 4.90 \times 10^{27}$ g
$R_{\rm Jupiter} = 7.14 \times 10^{4}$ km	$R_{\rm Venus} = 6.05 \times 10^{3}$ km
$H_0 = 100h$ km s^{-1} Mpc^{-1}	$T = (0.290\ {\rm cm})/\lambda_{\rm max}$ K
$\tau^2 = \dfrac{4\pi^2}{G} \dfrac{a^3}{M_1 + M_2}$	1 W = 1 J s^{-1} = 10^7 erg s^{-1}

1. (25 points) White Dwarfs

Numerical modeling suggests that the recurrent classical nova U Scorpii is a system containing a 1.5 M_{Sun} subgiant star in orbit about a 1.37 M_{Sun} white dwarf. As mass from the subgiant star accumulates on the surface of the white dwarf, it forms a thin, dense atmosphere around the white dwarf.

a) Assuming that the energy released during a nova outburst (about 3×10^{39} J) is entirely due to the fusion of hydrogen nuclei to helium in the white dwarf atmosphere, determine the mass of hydrogen (in units of M_{Sun}) that must fuse to produce the observed energy output.

b) Compute the surface gravitational acceleration g of the white dwarf (neglect the contribution of the atmosphere). Assume that the radius of a 1.37 M_{Sun} white dwarf is $R_{\text{WD}} \approx 2 \times 10^6$ m.

c) Assume that the atmosphere is plane-parallel, in hydrostatic equilibrium, and that it consists of nondegenerate, fully ionized hydrogen at constant temperature $T = 10^5$ K. Also assume that the gravitational acceleration is independent of height in the atmosphere. Under these assumptions, find an expression for the atmospheric density ρ as a function of height z above the surface. Obtain a numerical value for the atmospheric scale height h.

2. (25 points) Core-Collapse Supernovae

The Type II supernova SN 1987a occurred when the star Sk -69 202 underwent core collapse. Sk -69 202 was a blue supergiant; based on its luminosity and spectral class, it would have had a radius of about 4×10^{10} m.

a) A burst of neutrinos was observed by three different detectors approximately three hours before visible light from the supernova was seen. Explain this time delay and use it to estimate the supernova shock speed inside the star's envelope.

b) Assume that 1% of the supernova's energy of 5×10^{46} J went into kinetic energy of the stellar envelope, and also that the envelope's binding energy per unit mass at the time of the explosion was -1×10^{13} J kg^{-1}. If the envelope was ejected at the speed calculated in part (a), estimate the mass of the envelope in solar masses.

c) UV light from SN 1987a has ionized a circular ring of circumstellar gas centered on the explosion, causing it to glow because of recombination. This ring is inclined 42° to the line of sight, so it appears elliptical in the plane of the sky, with a major axis measuring 1.66 arcsec. Light was received from the near side of the ring about 360 days before light from the far side. Using this information, estimate the distance to SN 1987a in kpc.

3. (25 points) Basic Stellar Properties

The second brightest star in the sky, Canopus, has an absolute bolometric magnitude of -5.39 and an effective temperature of 7280 K. Its parallax distance as determined by the Hipparcos satellite is 94.8 pc.

a) Determine the luminosity of Canopus in units of the Sun's luminosity, given the Sun's absolute bolometric magnitude of 4.75.

b) Estimate the radius of Canopus in units of the Sun's radius, given the Sun's effective temperature of 5,778 K.

c) Using the main-sequence mass-luminosity relation ($L \propto M^{3.6}$), find the ratio of the main-sequence lifetime of Canopus and that of the Sun.

d) The Sun is about 70% hydrogen by mass. Assuming that the Sun will fuse 10% of its hydrogen mass into helium over its main-sequence lifetime, estimate the numerical values in Myr for the main-sequence lifetimes of Canopus and the Sun.

4. (25 points) Stellar Interiors

Consider a star of solar composition ($X = 0.70$, $Y = 0.28$, $Z = 0.02$) with a density profile given by

$$\rho(r) = \rho_c \left[1 - \left(\frac{r}{R_*}\right)^2\right],$$

where ρ_c is the central density and R_* is the radius of the star.

a) Calculate the mass $M(r)$ inside a radius r in terms of R_* and the total mass of the star, M_*.

b) Assuming that the star is in hydrostatic equilibrium and that the surface pressure $P(R_*)$ vanishes, determine the central pressure P_c in terms of M_* and R_*.

c) Find an expression for R_* in terms of M_* and the central temperature T_c. Give a numerical result in "canonical astrophysical form" with M_* expressed in units of M_{Sun} and T_c expressed in units of 10^6 K. (E.g., in canonical form the expression for the Schwarzschild radius is $R_S = 3.0\ (M/M_{Sun})$ km.) When computing the mean particle mass μ, assume that the gas is completely ionized.