ASTRONOMY PROFICIENCY EXAMINATION
STELLAR ASTROPHYSICS
August 24, 2012, 1:00 – 3:00 PM

INSTRUCTIONS:

- Answer ALL questions.
- Read a problem carefully. Pause and think before you attempt to solve it.
- Always obtain an *algebraic* answer first; then and only then an arithmetic one. BUT: if asked for, numbers are just as important as algebraic answers; do not skip them!
- Specify your units. An *arithmetic* answer without units is a *wrong* answer.
- Do NOT rename symbols given to you in a problem.
- Explain *briefly* what you are doing; that way you *may* get credit even if your solution is wrong. *If a grader cannot understand your reasoning, he/she may give you no credit.*
- Write LEGIBLY; illegible answers receive NO credit.
- You may use the back of the exam pages for scratch work.

TABLE OF USEFUL CONSTANTS AND SOME FORMULAE:

$M_{\rm Sun} = 1.99 \times 10^{33}$ g	$G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}
$R_{\rm Sun} = 6.96 \times 10^{10}$ cm	1 pc = 3.09×10^{18} cm
$\mathcal{L}_{\rm Sun} = 3.83 \times 10^{33}$ erg s^{-1}	1 yr = 3.16×10^{7} s
1 AU = 1.50×10^{13} cm	$c = 3.00 \times 10^{10}$ cm s^{-1}
$k_B = 1.38 \times 10^{-16}$ erg K^{-1}	$h = 6.63 \times 10^{-27}$ erg s
$m_p = 1.673 \times 10^{-24}$ g	$m_e = 9.11 \times 10^{-28}$ g = 0.511 MeV/c^2
$m_n = 1.675 \times 10^{-24}$ g	$m_\alpha = 6.6445 \times 10^{-24}$ g
$e = 4.80 \times 10^{-10}$ esu = 1.60×10^{-19} C	1 eV = 1.60×10^{-12} erg
$\sigma_{SB} = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}	1 J = 9.87×10^{-3} L · atm = 10^7 erg
1 m = 10^2 cm = 10^6 μm = 10^9 nm	1 Å = 10^{-8} cm = 10^{-10} m = 0.1 nm
$M_{\rm V(Sun)} = +4.83$ mag	$M_{\rm B(Sun)} = +5.48$ mag
$T_{\rm Sun} = 5{,}778$ K	$m_{\rm V(Sun)} = -26.5$ mag
$R_{\rm Earth} = 6.37 \times 10^{3}$ km	$R_{\rm Moon} = 1.74 \times 10^{3}$ km
$M_{\rm Earth} = 5.97 \times 10^{27}$ g	$r_{\rm Earth\text{-}Moon} = 3.84 \times 10^{5}$ km
$M_{\rm Jupiter} = 1.90 \times 10^{30}$ g	$M_{\rm Venus} = 4.90 \times 10^{27}$ g
$R_{\rm Jupiter} = 7.14 \times 10^{4}$ km	$R_{\rm Venus} = 6.05 \times 10^{3}$ km
$H_0 = 100h$ km s^{-1} Mpc^{-1}	$T = (0.290\ {\rm cm})/\lambda_{\max}$ K

$$\tau^2 = \frac{4\pi^2}{G}\,\frac{a^3}{M_1 + M_2} \qquad \frac{dP_{\rm rad}}{dr} = -\frac{\kappa\rho L}{4\pi r^2 c} \qquad 1\ {\rm W} = 1\ {\rm J\ s^{-1}} = 10^7\ {\rm erg\ s^{-1}}$$

1. (25 points) Eddington-Limited Accretion

Consider a star of mass M and radius R accreting mass onto its surface at a rate $\dot{M}$.

a) Assume that the accreting mass has the form of a spherical shell of ionized hydrogen gas at a distance r from the star, and that the opacity is κ. Find an expression for the luminosity L_{Edd} (the Eddington luminosity) at which the radiation force on the gas exactly balances the gravitational force on it.

b) Assuming that this accreting material releases 100% of its kinetic energy as radiation upon impact with the star's surface, find the accretion luminosity L_{acc}.

c) Determine the maximum rate $\dot{M}_{\text{Edd}}$ at which matter can accrete onto the star.

d) A neutron star is observed to accrete at a rate greater than the value determined in part (c). Give two possible explanations for this discrepancy.

2. (25 points) A Binary Star

A binary star system is observed using interferometry to have an orbital period of 2.31 yr and distances of the two stars from the system's center of mass (projected onto the sky) of 0.012" and 0.003".

a) Determine the ratio of the masses of the two stars.

b) Assume that the stars orbit in the plane of the sky. If the distance to the system is known to be 168 pc, determine the physical lengths of the semimajor axes of the orbits of the two stars in AU.

c) Again assuming the stars orbit in the plane of the sky, determine the sum of the masses of the two stars in solar masses (M_{Sun}).

d) Given your results from (a) and (c), determine the stars' masses in units of M_{Sun}.

3. (25 points) Stellar Interior

Consider a main-sequence star with a density profile for radii $r < R_*$ given by $\rho(r) = \rho_c(1 - r/R_*)$, where ρ_c is the central density and R_* is the radius of the star.

a) Find the enclosed mass profile $M(r)$ in terms of R_* and the total mass M_* of the star.

b) Assuming that the star is in hydrostatic equilibrium and that the surface pressure $P(R_*)$ vanishes, determine the central pressure P_c in terms of M_* and R_*.

c) Estimate the period of radial acoustic oscillations of the star in terms of M_*, R_*, and the adiabatic index γ of the gas.

4. (25 points) A Massive Star

During the last stages of a massive star's life, energy is generated in the stellar core by means of successive α captures onto silicon-28 nuclei. The α particles come from photodisintegration of silicon-28. The end product of these reactions is primarily nickel-56 (which later decays to form iron-56). Thus we can write roughly

$$2\,^{28}\text{Si} \rightarrow \,^{56}\text{Ni} + \text{energy}\,.$$

The atomic masses of ^{28}Si and ^{56}Ni are 4.645677×10^{-26} kg and 9.289408×10^{-26} kg, respectively.

a) Compute the fraction of the rest energy of the reactants that is released by this chain of reactions.

b) In a 22 M_{Sun} star nearing the end of its life, the core at the onset of silicon burning has mass of about 1.3 M_{Sun}. Estimate the total energy that can be released in the core by silicon burning. (Ignore any energy release due to contraction or other reactions.)

c) Suppose that the core could be extracted from the star. Treating the core as a uniform sphere of density 3.0×10^{10} kg m^{-3} and temperature 3.6×10^{9} K, estimate the luminosity of the extracted core during silicon burning.

d) Estimate how long the star will undergo Si burning. (Note that this will be shorter than the time for silicon burning inside the star since it does not account for energy radiated inward toward the core from the envelope.)